A CERTAIN INTEGRAL TRANSFORMS AND ITS PROPERTY

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ABSTRACT

The aim of present paper is to define some generalization results of K-transform by using chain of this transform. Some examples of the results are also given.

Keywords: *K-transform, General class of polynomial, Bessel function.*

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INTRODUCTION

If $g(y)$ and $f(x)$ are related by the integral equation

$$g(y) = \int_0^\infty f(x) k_v(xy) \sqrt{(xy)}\, dx \qquad (1.1)$$

Then $g(y)$ is said to be the K-transform of order v of $f(x)$ and regard y as a complex variable.

We shall denote (1.1.) symbolically as

$$g(y) = M^v[f(x)] \qquad (1.2)$$

This transform was introduced by Meijer [3] . Maheshwari [2] have studied the properties of the aforesaid transform by considering certain chains of this transform.

Srivastava [4] introduced the general class of polynomials (see also Srivastava and Singh [5])

$$S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} x^k,\ n = 0,1,2,... \qquad (1.3)$$

Where m and n are arbitrary integers the coefficients $A_{n,k} (n,k \geq 0)$ are arbitrary constants real or complex.

MAIN RESULTS

Theorem 1. If

$$M^v[f_1(x)] = g(y) \qquad (2.1)$$

$$M^{v}[f_2(x)S_n^m(\sqrt{x})] = \pi f_1\left(\frac{1}{y}\right) \quad (2.2)$$

Then

$$f(k)M^{2v}\left\{x^{k+\frac{3}{2}} f_2\left(\frac{x^2}{4}\right)\right\} = 4y^{\frac{3}{2}} g(y^2) \quad (2.3)$$

Provided $x^{\left(\pm v \pm k + \frac{1}{2}\right)} f_2(x)$ are bounded and absolutely integrable $(0,\infty)$ and $f(k) = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}$.

Further, let

$$M^{2v}\left[S_n^m(\sqrt{x}) f_3(x)\right] = \frac{\pi}{4} y^{-\frac{3}{2}} f_2\left(\frac{1}{4y^2}\right) \quad (2.4)$$

$$M^{2v}\left[S_n^m(\sqrt{x}) f_4(x)\right] = \frac{\pi}{4} y^{-\frac{3}{4}} f_3\left(\frac{1}{4y^2}\right) \quad (2.5)$$

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$$M^{2^{n-2}v}\left[S_n^m(\sqrt{x}) f_n(x)\right] = \frac{\pi}{4} y^{-\frac{3}{4}} f_{n-1}\left(\frac{1}{4y^2}\right) \quad (2.6)$$

Then

$$f(k)M^{2^{n-1}v}\left[x^{k+\frac{3}{2}} f_n\left(\frac{x^2}{4}\right)\right] = 4y^{\frac{3}{2}(2^{n-1}-1)} g\left(y^{2(n-1)}\right) \quad (2.7)$$

Provided $x^{\left(\pm 2^{n-1} v \pm k + \frac{1}{2}\right)} f_2(x)$ are bounded and absolutely integrable $(0,\infty)$ and $f(k) = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}$.

Proof: Taking $x^{\left(\pm 2^{n-2} v + k + \frac{1}{2}\right)} f_n(x), n = 2,3,\ldots,n$

Then by definition of K -transform, we obtain

$$M^{v}\left[f_1(z)\right] = \int_0^\infty f_1(z) k_v(zp)\sqrt{(zp)}dz$$

Write $f_1(z)$ from (2.2), we get

$$= \frac{1}{\pi}\int_0^\infty \left\{\int_0^\infty f_2(x)S_n^m(\sqrt{x})k_v\left(x/z\right)\sqrt{(x/z)}dx\right\} k_v(zp)\sqrt{(zp)}dz$$

Interchanging the order of integration which is justified under the conditions mentioned in the theorem and use the series representation of general class of polynomial, we get

$$=\frac{1}{\pi} f(k)\int_0^\infty \left\{\int_0^\infty \sqrt{p}\, k_v(zp)k_v\left(z/p\right)dz\right\} x^{\frac{k+1}{2}} f_2(x)dx$$

Now evaluating the inner integral by ([1], p.146), we get

$$=\frac{1}{\pi} f(k)\int_0^\infty \pi p^{-\frac{1}{2}} k_{2v}(2\sqrt{(xp)}\, x^{\frac{k+1}{2}} f_2(x)dx$$

Or

$$= g(y) = f(k)\int_0^\infty y^{-\frac{1}{2}} k_{2v}(2\sqrt{(ty)}\, t^{\frac{k+1}{2}} f_2(t)dt \qquad (2.8)$$

Writing $y = y^2$ and $t = \frac{t^2}{4}$, we obtain from (2.8)

$$4y^{\frac{3}{2}} g(y^2) = f(k)M^{2v}\left\{x^{k+\frac{3}{2}} f_2\left(\frac{x^2}{4}\right)\right\}$$

Proceeding successively, we assume the result (2.7).

Also let

$$\pi y^{-\frac{3}{2}} f_n\left(\frac{1}{4y^2}\right) = \int_0^\infty f_{n+1}(x) S_n^m(\sqrt{x}) k_{2^{n-1}v}(xy)\sqrt{(xy)}dx \qquad (2.9)$$

Substituting the expression for $f_n\left(\frac{x^2}{4}\right)$ from (2.9) in (2.7), interchanging the order of integration, using the series representation of general class of polynomial and evaluating the later integral by ([1], p.146), we obtain

$$y^{\frac{3}{2}(2^{n-1}-1)} g(y^{2^{n-1}}) = \frac{1}{\sqrt{y}} f(k)\int_0^\infty t^{k+\frac{1}{2}} f_{n+1}(t) k_{2^n v}(ty)\sqrt{(ty)}dt \qquad (2.10)$$

Writing $y = y^2$ and $t = \frac{t^2}{4}$, we obtain from (2.10)

$$y^{\frac{3}{2}(2^n-1)} g(y^{2^n}) = f(k)\int_0^\infty t^{k+\frac{3}{2}} f_{n+1}\left(\frac{t^2}{4}\right) k_{2^n v}(ty)\sqrt{(ty)}\, dt$$

i.e. $f(k)M^{2v}\left\{x^{k+\frac{3}{2}} f_{n+1}\left(\frac{x^2}{4}\right)\right\} = y^{\frac{3}{2}(2^n-1)} g(y^{2^n})$.

We thus find that if (2.7) is true for $n=2$, it is also true for $(n+1)$ i.e. for the next higher order. But we have seen that it is true for $n=2$ and so it is true for $n=3$ and so on. Hence (2.7) is true for all positive integral values of n except 1.

EXAMPLES ON THE THEOREMS

Example 1. Let

$$f_1(x)=\sqrt{\pi}\,2^{-v}a^{(2v-1)}x^{2v}J_{v-\frac{1}{2}}\left(\frac{a^2x}{2}\right)S_n^m(\sqrt{x})$$

Then making use of result ([1], p. 137), we obtain from (2.1)

$$g(y)=f(k)\frac{\sqrt{\pi}a^{(4v-2)}}{y^{\left(3v+k+\frac{1}{2}\right)}}\Gamma\left(2v+k+\frac{1}{2}\right)\left(1+\frac{a^2}{4y^2}\right)^{-2v-k-\frac{1}{2}}$$

$$\mathrm{Re}(v)>-\frac{1}{4},\mathrm{Re}(y)>\left|\mathrm{Im}\frac{a^2}{4}\right|.$$

From (2.2) and ([1], p. 148), we obtain

$$f_2(k)=\frac{x^{\left(v-k-\frac{1}{2}\right)}}{\pi}f(k)I_{2v}(a\sqrt{x})J_{2v-1}(a\sqrt{x})$$

$$\mathrm{Re}(v)>0,\mathrm{Re}(y)>0.$$

Taking $n=2$, we obtain from (2.7)

$$f(k)M\left\{\frac{x^{(2v+k+\frac{1}{2}}}{2^{(2v-1)}\pi}I_{2v-1}J_{2v-1}(ax/2)\right\}$$

$$=f(k)\frac{4\sqrt{\pi}a^{(4v-2)}}{y^{6v-k-\frac{1}{2}}}\Gamma\left(2v+k+\frac{1}{2}\right)\left(1+\frac{a^4}{4y^4}\right)^{-2v-\frac{1}{2}}$$

$$\mathrm{Re}(v)>0,\mathrm{Re}(y)>\mathrm{Re}(a/2).$$

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