ANALYSIS OF DIPERSION RELATION FOR HIGHER AZIMUTHAL PERIODICITY IN HELICALLY CORRUGATED WAVEGUIDE

Vasudha Rajput Thakur,

Assistant Professor,

M. B. Khalsa College Indore, India

ABSTRACT

In this paper, the qualitative characteristics of dispersion relation for a helically corrugated waveguide are discussed with numerical approach which is based on solution of dispersion equation derived from theory. It imposes certain restrictions allowed for the values of the axial and azimuthal symmetry of the structure.

The last section deals with results and discussion for different corrugation depth and azimuthal periodicity of the structure.

INTRODUCTION

Metal hollow waveguides with various types of periodic corrugation are widely used in high-power microwave electronics. One such structure that has recently attracted considerable interest consists of a helical corrugation in the wall of a circular cylindrical waveguide, which involves both axial and azimuthal periodicity. This provides asymmetry of the wave dispersion for circularly polarized modes, resulting in additional mode selection. These properties make waveguides with a helical corrugation attractive for a large number of applications.[7]

In particular, they have been used as slow-wave interaction structures in relativistic Cherenkov devices in Bragg and as mode converters etc. Helically corrugated waveguides have recently been successfully used as interaction regions in gyrotron traveling-wave tubes (GTWTs), and gyrotron backward-wave oscillators (GBWOs) and as a dispersive medium for passive microwave pulse compression [1-4].

Due to this wide applicability, it is relevant and important to investigate the electrodynamic properties of such waveguides by analytical approach and confirm the validity of the results by comparison with simulation and experimental studies. (5).

Helical corrugation of the inner surface of an oversized circular waveguide provides very flexible dispersion characteristic of an eigenwave. Under certain corrugation parameters, the eigenwave can possess a sufficiently high and almost constant group velocity over a wide frequency band in the region of close-to-zero axial wavenumber [6].

In previous paper [7], we derived the dispersion relation for a helically corrugated waveguide and discussed the result of dispersion for periodicity in azimuthal direction ϕ_1. where $q_0 = \frac{2\pi}{\varphi_1}$ q_0 =0, and 1.

Here the well known Floquet's theorem is used which deals with the eigenfunctions of wave equation in an infinite periodic structure. It imposes certain restrictions allowed by the axial and azimuthal symmetry of the structure.[7]

The principles of synthesizing the necessary dispersion and its qualitative characteristics are discussed with numerical approach which is based on solution of a dispersion equation derived from the theory [7]. In this paper necessary qualitative characteristic are discussed for $q_0 = 3$ with numerical approach and compared to other values of q_0 .The last section deals with results and discussion.

FIELDS IN HELICAL WAVEGUIDE

Let us consider a waveguide with the helical profile of its inner surface represented in a cylindrical coordinate system (*r, ɵ, z)* as follows:

Wall radius $R(z,\theta) = R_0 + h\cos(k_0 z + q_0\theta)$ →(1)

Where $k_0 = \frac{2\pi}{z_0}$ and $q_0 = \frac{2\pi}{\varphi_1}$. Periodicity in axial direction is z_0 while periodicity in azimuthal direction is ϕ_1.

Here R_0 is the mean radius of the waveguide, *h* is the amplitude of the corrugation, and z_0 is the corrugation period.

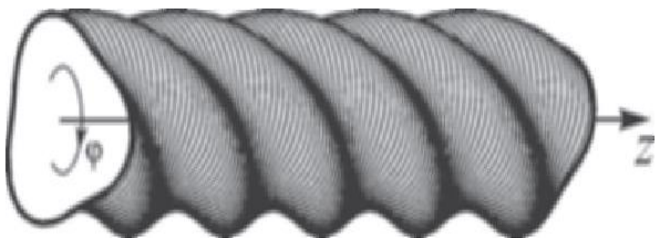


Figure.1 Schematic **view of a waveguide with a three-fold right-handed (*q_0*=3) helical corrugation.**

Due to Floquet periodicity both in azimuthal as well as axial direction, we can write the azimuthal as well as axial fields as

$$\vec{E} = \sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{e^{i\varphi_{np}}}{\beta_n^2}\begin{bmatrix} ik_{zn}\beta_n \frac{\partial}{\partial\beta_n r}J_{l_p}(\beta_n r)A_{np} & -\frac{l_p k}{r}J_{l_p}(\beta_n r)D_{np} \\ -\frac{l_p k_{zn}}{r}J_{l_p}(\beta_n r)A_{np} & -ik\beta_n\frac{\partial}{\partial\beta_n r}J_{l_p}(\beta_n r)D_{np} \\ \beta_n^2 J_{l_p}(\beta_n r)A_{np} & \end{bmatrix} \text{ and (2)}$$

$$\vec{B} = \sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{e^{i\varphi_{np}}}{\beta_n^2}\begin{bmatrix} \frac{l_p k}{r}J_{l_p}(\beta_n r)A_{np} & ik_{zn}\beta_n\frac{\partial}{\partial\beta_n r}J_{l_p}(\beta_n r)D_{np} \\ ik\beta_n\frac{\partial}{\partial\beta_n r}J_{l_p}(\beta_n r)A_{np} & -\frac{l_p k_{zn}}{r}J_{l_p}(\beta_n r)D_{np} \\ & \beta_n^2 J_{l_p}(\beta_n r)D_{np} \end{bmatrix} \rightarrow (3)$$

where

$$\varphi_{np} = -(\omega t - k_{zn}z - l_p\theta), \quad k_{zn} = k_z + nk_0 \quad and \quad l_p = l + pq_0$$

Here N is an integer tending to infinity.

BOUNDARY CONDITIONS

The tangential component of electric field must vanish at the metallic surface.

This result in

$$E_z - hk_0 \sin(k_0 z + q_0\theta)E_r \Big|_{R(z,\theta)=R_{00}+h\cos(k_0 z+q_0\theta)} = 0 \rightarrow (4)$$

And

$$E_\theta + \alpha\cos(k_0 z + q_0\theta)E_\theta - q_0\alpha\sin(k_0 z + q_0\theta)E_r \Big|_{R(z,\theta)=R_0+h\cos(k_0 z+q_0\theta)} = 0 \rightarrow (5)$$

here $\alpha = \dfrac{h}{R_0}$ (6)

Substituting electric field components E_r, E_θ from (6) in the first and second boundary conditions we get

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{e^{i\phi_{np}}}{\beta_n^2}\left\{\beta_n^2 J_{l_p}(\beta_n R)A_{np} - hk_0\sin(k_0 z+q_0\theta)\left[ik_{zn}\beta_n\frac{\partial}{\partial\beta_n r}J_{l_p}(\beta_n r)\Big|_R A_{np} - \frac{l_p k}{R}J_{l_p}(\beta_n R)D_{np}\right]\right\} \equiv 0 \rightarrow (7a)$$

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N} -\frac{e^{i\Phi_{np}}}{\beta_n^2}\left\{\begin{array}{l}\dfrac{[1+\alpha\cos(u')]}{2}\left[k_{zn}\beta_n\left(J_{l_p-1}+J_{l_p+1}\right)A_{np} + ik\beta_n\left(J_{l_p-1}-J_{l_p+1}\right)D_{np}\right] + \\ \dfrac{q_0\alpha}{2}\sin(u')\begin{bmatrix} ik_{zn}\beta_n\left(J_{l_p-1}-J_{l_p+1}\right)A_{np} \\ -k\beta_n\left(J_{l_p-1}+J_{l_p+1}\right)D_{np}\end{bmatrix}\end{array}\right\} = 0 \rightarrow (7b)$$

We drop the common factor – *(ωt –kzz–lϑ)* from $\varphi_{np} = -(\omega t - k_{zn}z - l_p\theta)$,

and substituting $k_0 z = u,\quad q_0\theta = \delta \quad and \quad u+\delta = u'.$

Also dropping the argument $\beta_n r$ from Bessel functions, we get

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{e^{i(nu+p\delta)}}{\beta_n^2}\left\{\beta_n^2 J_{l_p}A_{np} - \frac{hk_0\beta_n}{2}\sin(u')\left[ik_{zn}\left(J_{l_p-1}-J_{l_p+1}\right)A_{np} - k\left(J_{l_p-1}+J_{l_p+1}\right)D_{np}\right]\right\} = 0 \rightarrow (8)$$

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N} -\frac{e^{i(nu+p\delta)}}{\beta_n^2}\left\{\begin{array}{l}\dfrac{[1+\alpha\cos(u')]}{2}\left[k_{zn}\beta_n\left(J_{l_p-1}+J_{l_p+1}\right)A_{np} + ik\beta_n\left(J_{l_p-1}-J_{l_p+1}\right)D_{np}\right] + \\ \dfrac{q_0\alpha}{2}\sin(u')\begin{bmatrix} ik_{zn}\beta_n\left(J_{l_p-1}-J_{l_p+1}\right)A_{np} \\ -k\beta_n\left(J_{l_p-1}+J_{l_p+1}\right)D_{np}\end{bmatrix}\end{array}\right\} = 0 \rightarrow (9)$$

DISPERSION RELATION

Rewriting equations (8) and (9) and multiplying $e^{i(mu+o\delta)}$, and we get

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{e^{i[(n-m)u+(p-o)\delta]}}{\beta_n^2}\left\{\beta_n^2 J_{l_p}A_{np}-\frac{hk_0\beta_n}{2}\sin(u')\left[ik_{zn}\left(J_{l_p-1}-J_{_p l+1}\right)A_{np}-k\left(J_{l_p-1}+J_{l_p+1}\right)D_{np}\right]\right\}=0 \rightarrow \textbf{(10)}$$

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N}-\frac{e^{i[(n-m)u+(p-o)\delta]}}{\beta_n^2}\left\{\begin{matrix}\frac{\left[1+\alpha\cos(u')\right]}{2}\left[k_{zn}\beta_n\left(J_{l_p-1}+J_{l_p+1}\right)A_{np}+ik\beta_n\left(J_{l_p-1}-J_{l_p+1}\right)D_{np}\right]+\\ \frac{q_0\alpha}{2}\sin(u')\left[\begin{matrix}ik_{zn}\beta_n\left(J_{lp-1}-J_{l_p+1}\right)A_{np}\\-k\beta_n\left(J_{l_p-1}+J_{l_p+1}\right)D_{np}\end{matrix}\right]\end{matrix}\right\}=0 \rightarrow \textbf{(11)}$$

Integrate above equations over u in (limit –π to π)

We evaluate the integral

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{4\pi}{\beta_n^2}\left\{\beta_n^2 J_{l_p}^{nm}A_{np}+\frac{ihk_0\beta_n}{4}\left[\begin{matrix}ik_{zn}\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}-L_{l_p}^{nm-1}+L_{l_p}^{nm+1}\right)A_{np}\\-k\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}-L_{l_p}^{nm-1^1}-L_{l_p}^{nm+1}\right)D_{np}\end{matrix}\right]\right\}\delta_{p-o-n+m,0}=0$$

$$\sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{4\pi}{\beta_n^2}\left\{\begin{matrix}\left[\beta_n^2 J_{l_p}^{nm}-\frac{hk_0\beta_n}{4}k_{zn}\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}-L_{l_p}^{nm-1}+L_{l_p}^{nm+1}\right)\right]A_{np}\\-\frac{ihk_0k\beta_n}{4}\left[\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}+L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)\right]D_{np}\end{matrix}\right\}\delta_{p-o-n+m,0}=0 \rightarrow \textbf{(12)}$$

Similarly integrating equation (11) gives

$$-2\pi\sum_{n=-N}^{N}\sum_{p=-N}^{N}\frac{1}{\beta_n^2}\left\{\begin{matrix}\left[\begin{matrix}k_{zn}\beta_n\left(H_{l_p}^{nm}+L_{l_p}^{nm}\right)+\frac{\alpha k_{zn}\beta_n}{2}\left(H_{l_p}^{nm-1}+H_{l_p}^{nm+1}+L_{l_p}^{nm-1}+L_{l_p}^{nm+1}\right)\\+\frac{q_0\alpha}{2}k_{zn}\beta_n\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}-L_{l_p}^{nm-1}+L_{l_p}^{nm+1}\right)\end{matrix}\right]A_{np}\\+i\left[\begin{matrix}k\beta_n\left(H_{l_p}^{nm}-L_{l_p}^{nm}\right)+\frac{\alpha k\beta_n}{2}\left(H_{l_p}^{nm-1}+H_{l_p}^{nm+1}-L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)\\+\frac{q_0\alpha}{2}k\beta_n\left(H_{l_p}^{nm-1}-H_{l_p}^{nm+1}+L_{l_p}^{nm-1}-L_{l_p}^{nm+1}\right)\end{matrix}\right]D_{np}\end{matrix}\right\}\delta_{p-o-n+m,0}=0 \rightarrow \textbf{(13)}$$

Equations (12) and (13), will give the dispersion relation

We denote

$$i=(2N+1)[N+m]+N+o+1$$
$$j=(2N+1)[N+n]+N+p+1,$$

Then (12) and (13) can be written as

$$\sum_{j=1}^{(2N+1)(2N+1)} \left(T_{ij} G_j + U_{ij} K_j\right) = 0 \rightarrow \text{(14a)}$$

And

$$\sum_{j=1}^{(2N+1)(2N+1)} \left(V_{ij} G_j + W_{ij} K_j\right) = 0 \rightarrow \text{(14b)}$$

Where

$$T_{ij} = \left[J_{l_p}^{nm} - \frac{hk_0 k_{zn}}{4\beta_n} \left(H_{l_p}^{nm-1} - H_{l_p}^{nm+1} - L_{l_p}^{nm-1} + L_{l_p}^{nm+1} \right) \right] \delta_{p-o-n+m,0} \rightarrow \text{(15a)}$$

$$U_{ij} = \left[-\frac{ihkk_0}{4\beta_n} \left(H_{l_p}^{nm-1} - H_{l_p}^{nm+1} + L_{l_p}^{nm-1} - L_{l_p}^{nm+1} \right) \right] \delta_{p-o-n+m,0} \rightarrow \text{(15b)}$$

$$V_{ij} = \frac{k_{zn}}{\beta_n} \begin{bmatrix} \left(H_{l_p}^{nm} + L_{l_p}^{nm} \right) + \frac{\alpha}{2} \left(H_{l_p}^{nm-1} + H_{l_p}^{nm+1} + L_{l_p}^{nm-1} + L_{l_p}^{nm+1} \right) + \\ \frac{q_0 \alpha}{2} \left(H_{l_p}^{nm-1} - H_{l_p}^{nm+1} - L_{l_p}^{nm-1} + L_{l_p}^{nm+1} \right) \end{bmatrix} \delta_{p-o-n+m,0} \rightarrow \text{(15c)}$$

$$W_{ij} = \frac{ik}{\beta_n} \begin{bmatrix} \left(H_{l_p}^{nm} - L_{l_p}^{nm} \right) + \frac{\alpha}{2} \left(H_{l_p}^{nm-1} + H_{l_p}^{nm+1} - L_{l_p}^{n\ m-1} - L_{l_p}^{nm+1} \right) \\ + \frac{q_0 \alpha}{2} \left(H_{l_p}^{nm-1} - H_{l_p}^{nm+1} + L_{l_p}^{nm-1} - L_{l_p}^{nm+1} \right) \end{bmatrix} \delta_{p-o-n+m,0} \rightarrow \text{(15d)}$$

$$\begin{bmatrix} T_{ij} & U_{ij} \\ V_{ij} & W_{ij} \end{bmatrix} \begin{bmatrix} G_j \\ K_j \end{bmatrix} = 0 \rightarrow \text{(16)}$$

For non trivial solution the determinant of the matrix must be zero.

The required cold dispersion relation for helically corrugated waveguide can be given by the following matrix.

$$D(\omega, k) = \det \begin{bmatrix} T_{ij} & U_{ij} \\ V_{ij} & W_{ij} \end{bmatrix} = 0 \rightarrow \text{(17)}$$

in order to have non-zero G_j and K_j in (17).

RESULTS AND DISCUSSION

In this section we examine the operation of helically corrugated waveguide on the basis of numerical results obtained on the basis of above analytical results. Dispersion curves are obtained for the following set of parameters:

Mean radius of the waveguide $R_0 = 1.47cm$

Corrugation period $z_0 = 2.89$, l=1

$q_0 = 3$ corrugation depth h= 0.19cm, 0.175cm, 0.14cm, 0.1cm, 0.01cm.

$q_0 = 2$, corrugation depth h= 0.19cm, 0.175cm, 0.14cm, 0.1cm, 0.01cm.

In the subsequent computation, we limit ourselves to the case of (l=0 and l=1) for simplicity. To clarify the nature of (16), we consider the case without corrugation. Putting $h = 0$ and $q_0 = 0$,it results in the conventional dispersion relation. When the amplitude of corrugation decrease, the dispersion relation in the helically corrugated waveguide become close to those of the smooth waveguide as shown in Figure 2.

Dispersion curves for l=1 is numerically obtained from (17) for the given set of corrugation depth. The rank of determinant in (17) is infinite, and we have to approximate the determinant with an adequate finite rank in numerical analysis. In our numerical calculation, we consider N=2 the determinant of the order 50×50 is obtained.

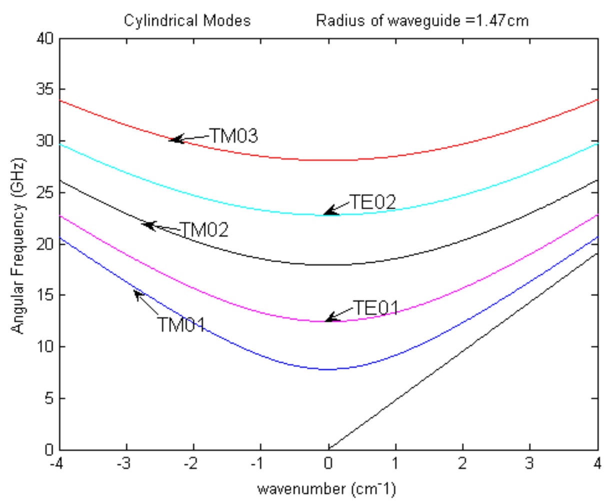
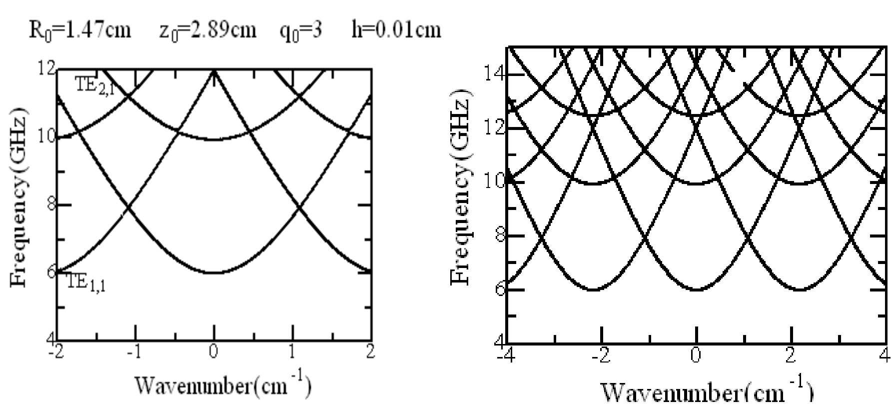


Figure .2 Dispersion characteristics of a cylindrical smooth waveguide of radius 1.47cm and light line.

Figure 2 shows TM and TE modes in cylindrical smooth waveguide with radius 1.47cm. The figure is drawn so that modes can be identified in the subsequent figures.

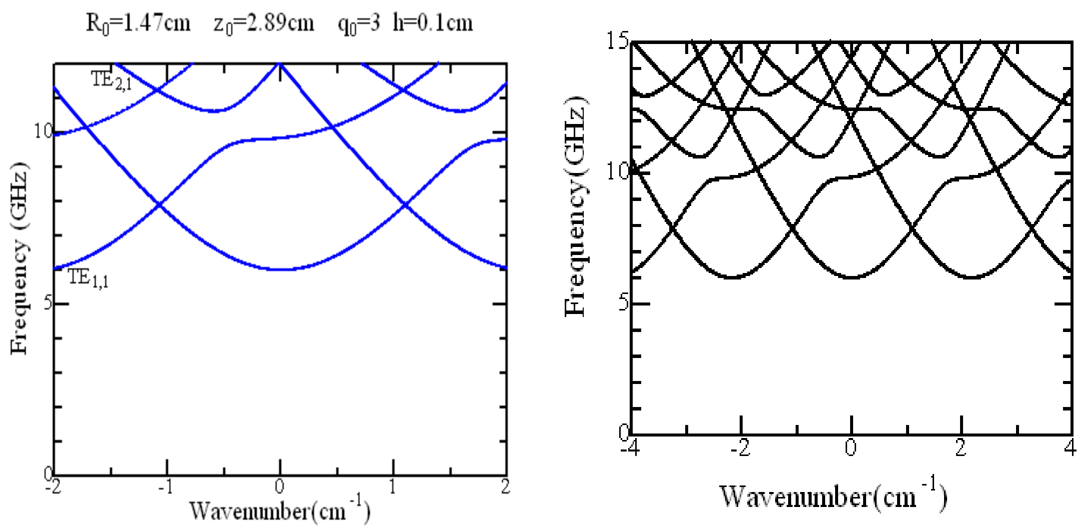
Figure 3 represent the dispersion relation of radius 1.47cm, corrugation period 2.89 and the lowest value of corrugation amplitude 0.01cm among all calculated results.TE_{11} and TE_{21} are shown. As the corrugation amplitude is very small the characteristics are similar to the cylindrical smooth waveguide.



(a) (b)

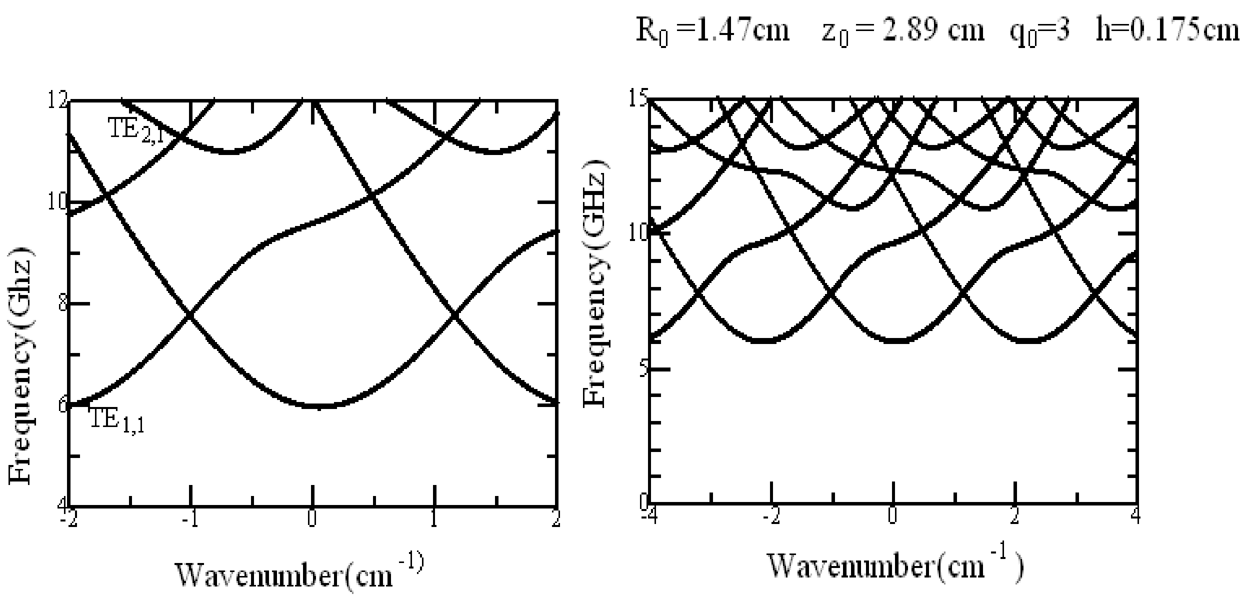
Figure.3(a) Dispersion characteristics for the helically corrugated waveguide of corrugation amplitude 0.01cm.

Figure 4 shows dispersion relation for corrugation amplitude of 0.1cm. Coupling of TE11 and TE_{21} are observed.



(a) **(b)**

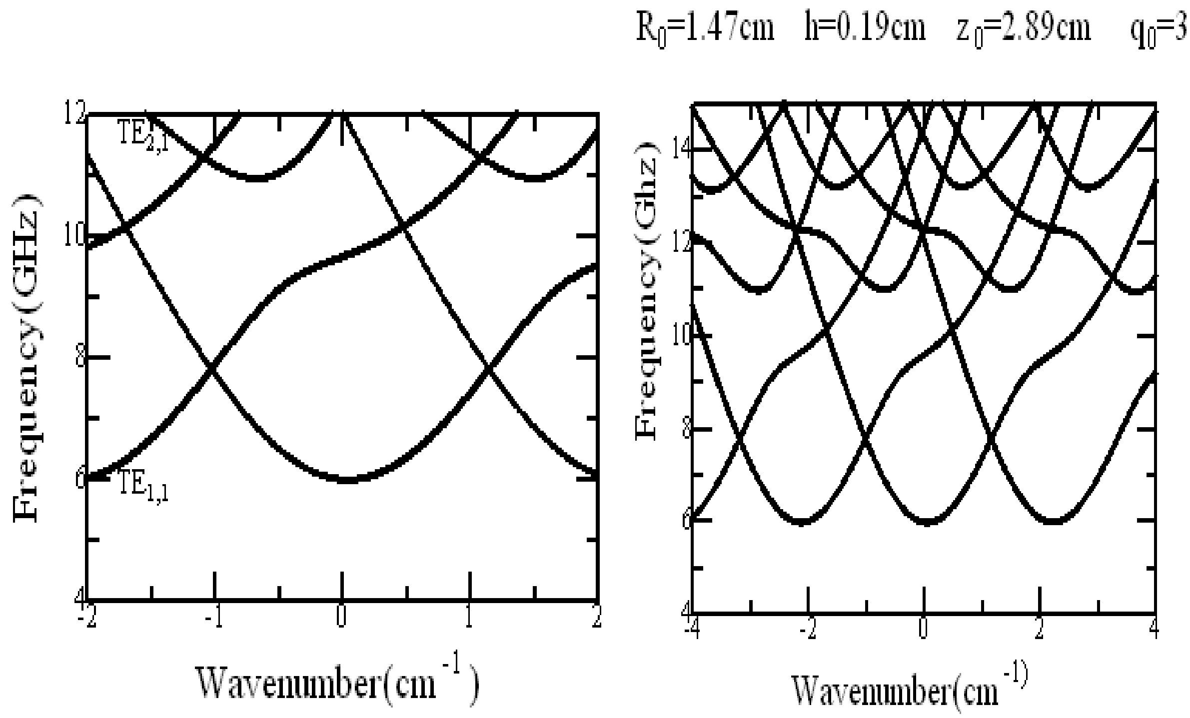
Fig. 4(a) Dispersion characteristics for the helically corrugated waveguide of corrugation amplitude 0.1cm.



(a) **(b)**

Figure. 5(a) Dispersion characteristics for the helically corrugated waveguide of corrugation amplitude 0.175cm. Coupling of TE_{11} mode and TE_{21} modes is evident (b).

Figure 5 and 6 present dispersion curve for corrugation amplitude 0.175cm and 0.19cm respectively where the Coupling of TE_{11} and TE_{21} are evident.



(a) (b)

Figure.6 (a) Dispersion characteristics for the helically corrugated waveguide of corrugation amplitude 0.19cm. The coupling of TE11 mode and TE21 modes is evident.

Figure 5 and 6 present dispersion curve for corrugation amplitude 0.175cm and 0.19cm respectively where the Coupling of TE_{11} and TE_{21} are evident

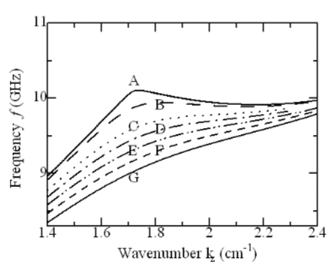


Figure.7 (a) Comparision of numerical results of dispersion curves for different value of corrugation amplitude. Here curves A, B, C, D, E, F and G are of corrugation amplitudes 0.01, 0.05,0.1,0.125,0.15,0.175 and 0.20cm respectively.

In figure. 7 shows the variation of frequency for various value of corrugation amplitude. Larger values of corrugation results in stronger coupling of TE_{11} and TE_{21} modes.

A helical wall perturbation can provide selective coupling between a higher and lower circularly polarized mode. With appropriate choice of parameters, the operating eigenwave of helically corrugated waveguide will be interpreted as the modified or strongly perturbed mode TE_{11}. The results obtained were compared with experimental and simulation which were found in good agreement **[1].**

REFERENCES

1. G. Burt, S. V. Samsonov] G. G. Denisov, A.R. Young, B.L. Bratman, A.D, R Phelps A. W. Cross, *Phys.Rev. E*, vol. 70,2004.
2. S. V. Samsonov, V. L. Bratman, G. G. Denisov, M. Y. Glyavin, V. K. Lygin, A.G. Luchinin, Y. V. Bykov, A. A. Bogdashov, and M. Thumm, *IEEE Trans.Plasma Sci.* vol.32, No. 3, June.2004.
3. G.S. Nusinovich, and O. Dhumdrajs, "Theory of gyro- backward wave oscillator with tapered magnetic field and waveguide cross section," IEEE Trans. Plasma Sci., vol.24, June 1996.
4. Wenlong He, K. Ronald, A.R. Young, A.W. Cross, A.D.R. Phelps, Colin.G. Whyte, E.D. Raffert, Thomson, C.W. Robertson, D.C. Speirs, S. V. Samsonov, V. L. Bratman, and G. G. Denisov, Gyro- BWO experiments using helical interaction waveguide," IEEE Trans. On Electron Devices. Vol. 52, No. 5 May 2005.
5. G.G. Denisov, V. L. Bratman, A. W. Cross, WelongHe, A.D. R. Phelps, and S.V. Samsonov, Gyrotron trvelling wave amplifier with a helical interaction waveguide," Phys.Rev. Letter. Vol. 81,1998.
6. G.G Denisov, V. L. Bratman, A. W. Cross, WelongHe, A.D. R. Phelps, and S.V. Samsonov, "High gain wide \- band gyrotron travelling wave amplifier with a helically corrugated waveguide," Phys.Rev. Letter. Vol. 84, March 2000.
7. Vasudha Rajput Thakur, "Cold Dispersion of Helically Corrugated Wave" IJRAR Vol.6 May 2019.
8. Graeme Burt, S. V. Samsonov] G. G. Denisov, V. L. Bratman, A. W. Cross, W. He, A.D. R. Phelps, K. Ronald, S.V. Samsonov, IEEE Trans. Plasma Sci., vol. 33, no.2, pp. 661–667April., 2005.
9. S. V. Samsonov *et al.*, Phys. Rev. Lett. vol. 92, p. 118 301, 2004.
10. R.A. Bromlry, B.E. Callan, proc. IEEE114 (9) 1213-1218. (1967).
11. S. V. Samsonov, V. L. Bratman, G. G. Denisov, M. Y. Glyavin, V. K. Lygin, A.G. Luchinin, Y. V. Bykov, A. A. Bogdashov, and M. Thumm, *IEEE Trans.Plasma Sci.* vol.32, No. 3, pp. 884–889, Dec.2004.
12. G.S. Nuinovich, Y. Carmel, T.M. Antonsen, D.M. Goebel and J. Santoru, IEEE Trans. Plasma Sci., vol. 26, No. 3, pp. 628-645, June 1998.
13. A.W. Trievelpiece and R.W. Gould, J.Appl. Phys. Vol.30. pp. 1784-1799,1959.
14. M.V. Kuzelev, O.T. Loza, A.A. Rukhadze, P.S. Strelkov and A.G. Shkvarunets, Plasma Phys. Reports, vol.2 No.8, pp. 669- 691, 2001.
15. D. Bohm and E. Gross, Phys. Rev; vol. 75, pp 1851- 1876,1949.
16. Y. Carmel, K. Minami, R. A Kehs, W. W. Destler, V.L. Granatstein, D. Abe, and W.L. Lou, Phys. Rev; vol.62, pp 389-2392, 1989.
17. Y. Carmel, W.R. Lou, T.M. Antonsen Jr., J. Rodgers, B. Levush, W.W. Destler, and

18. V.L. Granatstein, Phys.Fluids; vol.4, pp 2286-2292, 1992.
19. G. Shkvarunets, S. Kobayashi, J. Weaver, Y. Carmel, J. Rodgers, T.M. Antonsen Jr.,
20. V.L. Granastein, Destler, W. W.W "IEEE Transactions on Plasma Science vol. 24 pp. 905-917 June 1996
21. Preeti vyas, Arti Gkhale Y. Choyal, and K.P. Maheswari Pranma Journal of Ph ys. Indian academy of science.Vol.56 pp. 625-633 may 2001.and