EFFECTS OF SURFACE WAVINESS AS IN POISEUILLE FLOW

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ABSTRACT

In the problem I consider unsteady flow of a viscous incompressible fluid in a channel bounded by a lower plane surface and an upper wavy surface. The waviness in the surface is a function of time as well as axial distance. The complete Navier-Stokes equations are solved using regular perturbation method. The expressions for axial and normal velocities are derived and exhibited through graphs for different values of the parameters.

Keywords: *Poiseuille flow, surface waviness, pressure gradient.*

INTRODUCTION

The effects of axial roughness on velocity distribution for a fluid flowing in a tube or circular cylinder have been investigated by many researchers. Ralph A 6A studied the problem for oscillatory flows in wavy wall tubes for Reynolds numbers up to 300. Prakash A5A considered the slow unsteady flow of a viscous incompressible fluid in axisymmetric tube of varying radius. He obtained the series solution for stream function and pressure in powers of the small ratio of mean radius in characteristic length along the axes. Maha Lakshmi and DevanathanA4A studied the effects of buoyancy forces on the steady laminar viscous flow in a horizontal tube of varying cross section. LyneA3Aanalysed the unsteady viscous flow over a wavy wall. Gupta and Goyal A2A discussed the unsteady plane Poiseuille flow between two parallel plates. Duck A1A discussed the effects of small surface perturbations on the pulsatile boundary layer on a semi-infinite flat plate.

In the present paper I am considering the unsteady flow of a viscous incompressible fluid through a channel bounded by a lower plane surface and an upper wavy surface. The waviness in upper surface is varying with time as well as with axial distance and it is described by $y = a + \varepsilon \cos (w_1 t + w_2 x)$ where ε is small compared to mean distance '2a' between the surface. The flow in the channel is developed due to a constant pressure gradient in the x-direction. Regular perturbation technique has been applied to obtain the expression for axial and normal velocities. The obtained solutions have been numerically worked out for different values of the parameters.

PROBLEM FORMULATION

Consider the unsteady flow of a viscous incompressible fluid in a channel between two plates, the lower plate is taken as horizontal and upper plate is a wavy surface. We take x axis in the axial direction of the channel and y axis is perpendicular to it. Motion is two dimensional so all the variables will be independent of z coordinates. Taking origin in the middle point of channel. we take x-axis in the direction of fluid flow. The lower plate is situated at y = -a and upper wavy plate is situated at $y = a + \varepsilon \cos (w_1 t + w_2 x)$.

The Navier-Stokes equations for this flow reduce to

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ .... (1.1)

$\varrho\left(\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial y^2}\right)$ .... (1.2)

$\varrho\left(\frac{\partial v}{\partial t}+u\,\frac{\partial v}{\partial x}+v\,\frac{\partial v}{\partial y}\right)=\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$ .... (1.3)

and the boundary conditions are

$u=o\ at\ y=a+\varepsilon\cos\,(w_1t+\ w_2x)$ .... (1.4)

$u=o,v=o\ at\ y-\ -a)$

where 'a' be half height of the channel and ε is small compared to 'a' and u, v are the axial and normal velocities in the x and y direction, μ is the coefficient of viscosity.

Introducing the following non-dimensional quantities as :

$\bar{x}=\frac{x}{a}\,,\bar{y}=\frac{y}{a}\,,\bar{u}=\frac{ua}{\nu}\,,\bar{þ}=\frac{þa^2}{\nu}\,,\bar{t}=\frac{t\nu}{a^2}$

into the equations (1.1) to (1.4) and after dropping the bars, we obtain the following set of equations

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ .... (1.5)

$\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial þ}{\partial x}+\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ .... (1.6)

$\frac{\partial u}{\partial t}+u\,\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}$ ....(1.7)

The corresponding boundary conditions reduce to

$u=o\ at\ y=1+\varepsilon\cos\,(w_1t+\ w_2x)$

$u=o,v=o\ at\ y=\ -1$ .... (1.8)

where$\bar{w}_1=\frac{w_1a^2}{\nu}\,,\bar{w}_2=w_2a$

SOLUTION OF THE PROBLEM

Owing to the upper surface, we seek the solution of equations (1.6) - (1.8) in the form

$u=u_0(y)+\sum_{i=1}^{\infty}\varepsilon^i u_i\,(x,y,t)$

$v=v_0(y)+\sum_{i=1}^{\infty}\varepsilon^i v_i\,(x,y,t)$ ----------- (2.1)

whereu_0 and v_0 are the solutions when the surface is smooth.

Substituting (2.1) in (1.5) to (1.8) and comparing coefficients of like powers of ε, we obtain:

1st ε^0 *terms*

$\frac{\partial u_0}{\partial x}=0$ .... (2.2)

$\frac{\partial^2 u_0}{\partial y^2}=\frac{\partial p_0}{\partial x}=-A_0$ .... (2.3)

whereA_0 is constant pressure gradient in the x direction. The corresponding boundary conditions are

$u_0=o,at\ y=\ 1$

$u_0=o,v_0=o\ at\ y=-1$ .... (2.5)

The boundary conditions have been obtained by having Taylor's expansions of the boundary conditions at $y=1+\varepsilon\cos\,(w_1t+\ w_2x)$ and retaining terms independent of ε. We obtain the solution of (2.2) to (2.5) as:

$u_0=\frac{A_0}{2}\,(1-y^2)$ .... (2.6)

ε^0 *terms*

$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0$ .... (2.7)

$\frac{\partial u_1}{\partial x}+u_0\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_0}{\partial y}=\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}$ .... (2.8)

$\frac{\partial v_1}{\partial t}+u_0\frac{\partial v_1}{\partial x}=\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial y^2}$ .... (2.9)

and the boundary conditions reduce to

$u_1=A_0\cos\,(w_1t+\ w_2x)\ at\ y=1$

$u_1=o,v_1=o\ at\ y=-1$ .... (2.10)

To solve these equations (2.7) to (2.9), let us introduce a stream function ψ such that

$u_1=-\frac{\partial\psi}{\partial x}\,,v_1=\frac{\partial\psi}{\partial x}$ .... (2.11)

then equation of continuity (2.7) is automatically satisfied. From (2.8) , (2.9) and (2.11) , we obtain

$-\psi_{xyt}+\psi_{xxxy}+\psi_{yyxy}=0$ .... (2.12)

Now let us take

$\psi = Real\ part\ e^{i(w_1 t + w_2 x)} \sum_{n,s} w_1^n w_2^s f_{ns}\ (y)$ .... (2.13)

Substituting (2.13) into (2.12) and collecting the coefficients of like powers of w_1 *and* w_2, we obtain the following set of ordinary differential equations

$f_{00}''' = 0$ .... (2.14a)

$i\ f_{00}''' +\ f_{00}' = 0$ .... (2.14b)

$f_{01}''' = 0$ .... (2.14c)

$i\ f_{11}''' + f_{01}' = 0$ .... (2.14d)

$i\ f_{20}''' + f_{10}' = 0$ .... (2.14e)

$i\ f_{02}''' - i\ f_{00}' = 0$ .... (2.14f)

$i\ f_{21}''' +\ f_{11}' = 0$ .... (2.14g)

$i\ f_{22}''' - i f_{10}' +\ f_{02}' = 0$ .... (2.14h)

$i\ f_{22}''' + f_{12}' -\ i\ f_{20}' = 0$ .... (2.14i)

and corresponding boundary conditions are

$at\ y = 1\ ,\ f_{00}' = -A_0\ , f_{10}' = f_{01}' = f_{11}' = f_{12}' = f_{21}' = f_{20}' = f_{02}' = f_{22}' = 0$

$at\ y = -1 \qquad f_{00}' = f_{01}' = f_{10}' = f_{11}' = f_{12}' = f_{21}' = f_{20}' = f_{02}' = f_{22}' = 0$

$f_{00} = f_{01} = f_{10} = f_{11} = f_{12} = f_{21} = f_{20} = f_{02} = f_{22} = 0$ ......(2.15)

We obtain the solution from these equations (2.14a - 2.14i) under the boundary conditions (2.15)

RESULTS

The first order axial and normal velocity component i, e, u_1 and v_1 are shown in Fig. 1 and Fig. 2 respectively. The Fig. 1 indicates variation in u_1/A_0 for different values of $w_1 t + w_2 x$taking $w_1 = w_2 = a = 1$. Since in the expression of u_1/A_0 is guided by this term as evident in the figure. The perturbation part $v_1/A_0$is shown in Fig. 2. It is in the normal velocity phase difference of $\pi/2$ with axial component.

The composite axial velocity u is shown in fig. 3 for $\varepsilon = 0.1$ *and* $w_1 = w_2 = 1$. for the sake of comparison, the zeroth order axial velocity component u_0 is also shown there. It is observed that the maximum velocity occurs at y = 0 when these variation in 'u' due to surface waviness are more prominent for values of $y\ \varepsilon\ (0,1)$ compared to the value of $y\ \varepsilon\ (-1,0)$. This is in agreement with the physical situation also. Further we have $u > u_0$ at corresponding values of y for $w_1 t + w_2 x = 0, 2\pi$ *and* $u < u_0$ *for* $w_1 t + w_2 x = \pi$. Fig. 4 shows axial velocity component for different values of y in a complete wave.

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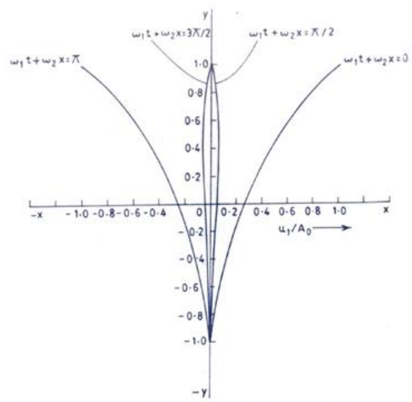

FIG.1 FIRST COMPONENT OF PERTURBATED AXIAL VELOCITY Vs y.

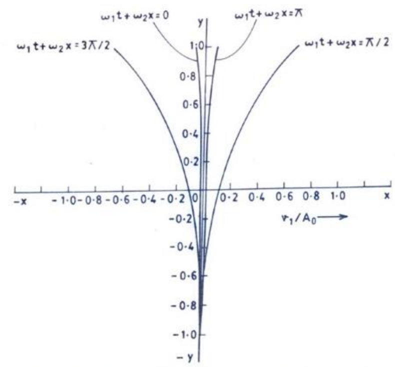

FIG.2 FIRST COMPONENT OF PERTURBATED NORMAL VELOCITY Vs y.

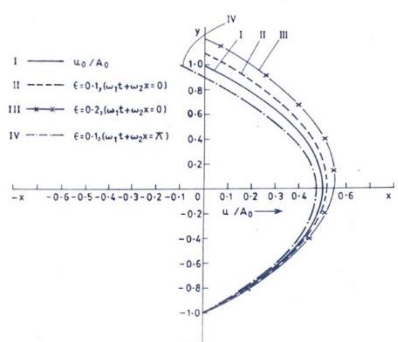

FIG.3 RADIAL VELOCITY Vs y.

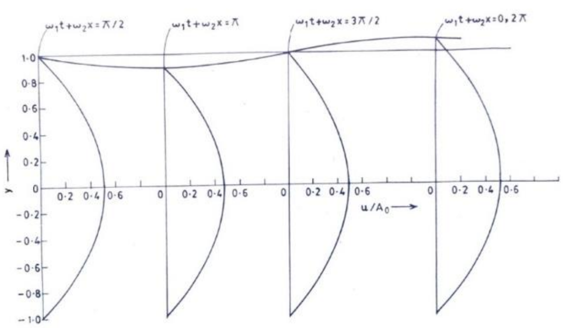

FIG.4 AXIAL VELOCITY Vs y.