FLOW PAST A CORRUGATED DISC IN A ROTATING SYSTEM

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ABSTRACT

The velocity and temperature fields are discussed in a rotating system of an in-compressible viscous fluid bounded by an infinite wavy porous disc with uniform suction or injection. The solutions are assumed to consists of a mean solution and a perturbed secondary solution. The obtained temperature and velocity distribution is graphically exhibited showing the impact of various parameters governing their distribution. It is observed that the maximum temperature occurs near the disc and decreases exponentially as we move away from the disc.

Keywords: *(Viscous flow/rotating system/corrugated disc/heat transfer)*

The study of the flow of an in-compressible or compressible viscous fluid bounded by wavy walls has been discussed by many researchers Lekondis and Nayfeh[4]. Effects of small amplitude wall waviness upon the stability of laminar boundary layer was studied by Lessem and Gangwani[5]. The problem of the flow of a viscous fluid due to impulsive motion of a wavy wall for low and high Reynolds number was studied by Shankar and Sinha[7]. They observed that at low Reynolds number the waviness of the wall quickly ceases to be of importance as the fluid is dragged alone by the wall, While for the high Reynolds numbers the effects of viscosity are confined in a thin boundary layer close to the wall. The structure of steady velocity field and the associate boundary frame of reference. He observed that thickness of boundary layer decreases due to the presence of suction. Purushothaman[6] discussed the fluctuating boundary layers in a rotating fluid with variable suction. Sharma[8] discussed unsteady M.H.D. flow in a porous medium past a oscillating plate in a rotating system.

In the present problem we have investigated the heat and mass transfer in a rotating flow of an in-compressible viscous fluid bounded by an infinite wavy porous disc. The plate and fluid are in a state of solid-state rotation, with a constant angular velocity about the z-axis normal to the plate. The bounding plate is considered to be corrugated Fig. 1. This perturbation in the surface gives rise to some perturbations in the flow and temperature distribution. The solutions are assumed to consist of (i) a mean, and (ii) a secondary perturbed part. These solutions are obtained using regular perturbation methods.

METHODS

We consider the flow over an infinite wavy porous plate lying in a vast expanse of viscous in - compressible fluid. The origin of the coordinate system is taken in the plate, z-axis being in the normal direction to the plate. The whole system is rotating as a solid body with constant angular velocity ν about z-axis. The waviness in the plate Fig. 1 is caused by circular grooves and the surface of the plate is described by z=$\varepsilon \cos \gamma (x^2+y^2)^{1/2}$. x-axis and y-axis are lying the plane of plate. For the flow under consideration the pressure gradient far away from the plate balances the Coriolis force. The fluid is assumed to be stationary, however, the plate is moving with a velocity - U* in its own plane. The flow is subjected to uniform suction or injection at the plate w = w_o , w_o> o for suction and w_o< o for blowing.

The equations governing the flow field and energy field are

$$u\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}-\;w_o\frac{\partial u}{\partial z}-2\,\Omega v=\;\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \quad \text{......... (1.1)}$$

$$u\frac{\partial v}{\partial x}+v\,\frac{\partial v}{\partial y}-\;w_o\frac{\partial v}{\partial z}-2\,\Omega u=\;\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \quad \text{......... (1.2)}$$

$$u\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y}-\;w_o\frac{\partial T}{\partial z}=\;\alpha_1\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \quad \text{......... (1.3)}$$

where (u, v, w) are the velocity components along x-axis, y-axis and z-axis.

We introduce a new variable $q=(u+iv)$ for combining (1.1) and (1.2) into a single equation.

$$u\frac{\partial q}{\partial x}+v\,\frac{\partial q}{\partial y}-\;w_o\frac{\partial q}{\partial z}+2\,\Omega\mathrm{iq}=\;\nu\left(\frac{\partial^2 q}{\partial x^2}+\frac{\partial^2 q}{\partial y^2}+\frac{\partial^2 q}{\partial z^2}\right) \quad \text{......... (1.4)}$$

Equation (1.3) and (1.4) are subjected to the boundary conditions.

$$q=\;-U^*,T=\;T_w,atz=\;\varepsilon^* cos\bar{y}(x^2+y^2)^{1/2}$$
$$q\rightarrow o,T\rightarrow T_\infty\;as\;z\;\rightarrow\infty \quad \text{......... (1.5)}$$

Using non-dimensional variables as defined below

$$\bar{z}=\frac{zU}{\nu}\;,\bar{q}=\;\frac{q}{U^*},\Theta=\frac{T-T_\infty}{T_w-T_\infty},\bar{x}=\;\frac{xU^*}{\nu},\bar{y}=\frac{yU^*}{\nu},$$
$s=\frac{w_o}{U^*}$ (suction parameter) , $R_{OT}=\frac{2\Omega\nu}{U^{*2}}$ (Rotational parameter)

$Pr=\frac{\mu\,C_p}{k}=\frac{\nu\ell C_p}{k}=\frac{\nu}{\alpha_1}$ (prandtl Number) .... (1.6)

In the equation of motion (1.4) and energy equation (1.3), we obtain the reduced equations after dropping the bars.

$$u\frac{\partial q}{\partial x}+v\,\frac{\partial q}{\partial y}-\;s\;\frac{\partial q}{\partial z}+1\;R_{OT}\mathrm{q}=\;\left(\frac{\partial^2 q}{\partial x^2}+\frac{\partial^2 q}{\partial y^2}+\frac{\partial^2 q}{\partial z^2}\right) \quad \text{......... (1.7)}$$

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}=Pr=\left(u\frac{\partial\theta}{\partial z}+v\frac{\partial\theta}{\partial y}-s\frac{\partial\theta}{\partial z}\right) \quad \text{......... (1.8)}$$

The ∞ responding boundary conditions are

$$q=-1,\theta=1\;at\;z=\varepsilon\;cos\gamma(x^2+y^2)^{1/2}$$
$$q\rightarrow o,\theta\rightarrow o\;as\;z\rightarrow\infty \quad \text{......... (1.9)}$$

where γ is the wall waviness parameter. We make use of perturbation methods to solve the equations (1.7) to (1.9), taking ε as the perturbation parameter. We assume the expansion in the form

$\phi = \phi_o(z) + \varepsilon\, \phi_1(x, y, z) + \varepsilon^2 \phi_2(x, y, z) + . \dots\dots$ .... (1.10)
where $\phi\ \equiv q, u, v, \theta$

Substituting the values of variables from (1.10) into (1.7) and (1.8) and ∞collecting the like powers of $\varepsilon^o, \varepsilon^1, \varepsilon^2 \dots \dots$ we obtain ε^o : zeroth order terms give

$$\frac{d^2\varrho_o}{dz^2} = -S\, \frac{d\varrho_o}{dz} + R_{OT}\, \varrho_o \quad \text{............ (1.11)}$$

$$\frac{d^2\varrho_o}{dz^2} = -S\, Pr \frac{d\phi_o}{dz} \quad \text{............. (1.12)}$$

ε^1 : First order terms give

$$\frac{\partial^2\varrho_1}{\partial x^2} + \frac{\partial^2\varrho_1}{\partial y^2} + \frac{\partial^2\varrho_1}{\partial z^2} = U_o \frac{\partial\varrho_1}{\partial x} + V_o \frac{\partial\varrho_1}{\partial y} - S \frac{\partial\varrho_1}{\partial z} + 1\, R_{OT}\varrho_1 \quad \text{........... (1.13)}$$

$$Pr\left[U_o \frac{\partial\phi_1}{\partial x} + V_o \frac{\partial\phi_1}{\partial y} - S \frac{\partial\phi_1}{\partial z}\right] = \frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_1}{\partial y^2} + \frac{\partial^2\phi_1}{\partial z^2} \quad \text{........... (1.4)}$$

ε^2: second order terms give

$$\frac{\partial^2\varrho_2}{\partial x^2} + \frac{\partial^2\varrho_2}{\partial y^2} + \frac{\partial^2\varrho_2}{\partial z^2} = U_o \frac{\partial\varrho_2}{\partial x} + V_o \frac{\partial\varrho_2}{\partial y} - U_1 \frac{\partial\varrho_1}{\partial x} + V_1 \frac{\partial\varrho_1}{\partial y} - S\, \frac{\partial\varrho_2}{\partial z} + 1\, R_{OT}\varrho_2 \quad \text{........ (1.15)}$$

$$Pr\left[U_o \frac{\partial\phi_2}{\partial x} + V_o \frac{\partial\phi_2}{\partial y} + U_1 \frac{\partial\varrho_1}{\partial x} + V_1 \frac{\partial\varrho_1}{\partial y} - S\, \frac{\partial\varrho_2}{\partial z}\right] = \frac{\partial^2\phi_2}{\partial x^2} + \frac{\partial^2\phi_2}{\partial y^2} + \frac{\partial^2\phi_2}{\partial z^2} \quad \text{.......... (1.16)}$$

The boundary conditions are

$$\varrho_o = -1, \phi_o = 1\ at\ z = 0$$
$$\varrho_o = , \phi_o \to 0\ at\ z \to \infty$$
$$\varrho_1(0) = -, \varrho_o(0) cos\, \gamma\, (x^2 + y^2)^{1/2}$$
$$\phi_1(0) = -, \phi_o(0) cos\, \gamma\, (x^2 + y^2)^{1/2}, \varrho_1\, , \phi_o \to 0\ at\ z \to \infty \quad \text{......... (1.17)}$$
$$\varrho_2(0) = -, \varrho_1(0) cos\, \gamma\, (x^2 + y^2)^{1/2} - \frac{\varrho_0(0)}{2} cos^2\, \gamma\, (x^2 + y^2)^{1/2}$$
$$\varrho_2(0) = -, \phi_1(0) cos\, \gamma\, (x^2 + y^2)^{1/2} - \frac{\phi_0(0)}{2} cos^2\, \gamma\, (x^2 + y^2)^{1/2}$$
$$\varrho_2\, , \phi_2 \to 0\ at\ z \to \infty \quad \text{........... (1.18)}$$

The solutions for the zeroth-order ϱ_o , ϕ_o obtained from (1.11) and (1.12) with the corresponding boundary conditions are

$$\varrho_o = -\, e^{-m_o z} \quad \text{............. (1.19)}$$

$$\varrho_o = -\, e^{-S\, \mathrm{Pr}\, Z} \quad \text{............. (1.20)}$$

Where

$$m_0 = \frac{s + \sqrt{S^2 + 41\, R_{OT}}}{2} \quad \text{............ (1.21)}$$

In order to solve (1.13) and (1.14) subject to (1.18), we assume

$$\varrho_1 = \varrho_1(z)\, e^{1\gamma(x^2+y^2)^{1/2}}$$

$$\phi_1 = \phi_1(z)\, e^{1\gamma(x^2+y^2)^{1/2}} \quad \text{............. (1.22)}$$

Substituting the values of ϱ_1, ϕ_1 in the equation (1.13) and (1.14) the reduced first terms differential equations are

$$\frac{\partial^2\varrho_1}{dz^2} + S\,\frac{\partial\varrho_1}{dz} + \varrho_1\left[1\gamma(x^2+y^2)^{-1/2}(1 - U_0x - V_0y) - 1\,R_{OT} - \gamma^2\right] = 0 \quad \text{........... (1.23)}$$

$$\frac{\partial^2\phi_1}{dz^2} + S\,\frac{\partial\phi_1}{dz} + \varrho_1\left[1\gamma(x^2+y^2)^{-1/2}(1 - U_0x\,Pr - V_0y\,Pr) - \gamma^2\right] = 0 \quad \text{........... (1.24)}$$

The coupled system of differential equations (1.23) and (2.24) are solved assuming $\gamma < 1$, for this we expand ϱ_1, ϕ_1 in a series of the form

$$[\varrho_1, \phi_1] = \overset{\varepsilon}{\underset{r=0,1,2\,.....}{}}\gamma^r[\varrho_{1r}, \phi_{1r}] \quad \text{.......... (1.25)}$$

Substituting (1.25) into (1.23) and (1.24) and comparing the like powers of 0, 1, 2...... we obtain following set of differential equations

$$\frac{\partial^2\varrho_{10}}{dz^2} + S\,\frac{\partial\varrho_{10}}{dz} - R_{OT}\varrho_{10} = 0 \quad \text{......... (1.26)}$$

$$\frac{\partial^2\varrho_{11}}{dz^2} + S\,\frac{\partial\varrho_{11}}{dz} - 1\,R_{OT}\varrho_{11} = -\varrho_{10}(x^2+y^2)^{-1/2}(1 - U_0x - V_0y) \quad \text{.........(1.27)}$$

$$\frac{\partial^2\phi_{10}}{dz^2} + S\,Pr\,\frac{\partial\phi_{10}}{dz} = 0 \quad \text{........ (1.28)}$$

$$\frac{\partial^2\phi_{11}}{dz^2} + S\,Pr\,\frac{\partial\phi_{11}}{dz} + 1(x^2+y^2)^{-1/2}\{1 - U_0x\,Pr - V_0y\,Pr\}\phi_{10} = 0 \quad \text{........ (1.29)}$$

$$\frac{\partial^2\phi_{12}}{dz^2} + S\,Pr\,\frac{\partial\phi_{12}}{dz} - \phi_{10} + 1(x^2+y^2)^{-1/2}\{1 - U_0x\,Pr - V_0y\,Pr\}\phi_{11} = 0 \quad \text{........ (1.30)}$$

The boundary conditions for ϱ_{1r} and ϕ_{1r} are as following :

$$\varrho_{10} = -\varrho_0\,,\phi_{10} = -\phi_0\,,\phi_{12} = \frac{\phi_0}{2}(x^2+y^2)at\ z = 0$$

$$\varrho_{11}\,,\varrho_{11} = 0\ at\ z = 0$$

$$at\ z \to \infty\,,\varrho_{10}\,,\varrho_{11}\,,\phi_{10}\,,\phi_{11}\,,\phi_{12}\,\dots\dots\dots \to 0 \quad \text{.......... (1.31)}$$

The solutions are

$$\varrho_{10} = -m_0e^{-m_0z} \quad \text{.......... (1.32)}$$

$$\varrho_{11} = c_{11}e^{-m_0 z} + 1m_0(x^2+y^2)^{-1/2}\left[\frac{e^{-m_0 z}}{S(S+\alpha+1\beta)} + e^{-k_1{}^2}\left\{\frac{\frac{\beta}{2}(S-2k_1)(x\, sin\frac{\beta z}{2+y}\cos\frac{\beta z}{2}) + L_1(x cos\,\frac{\beta z}{2-y}\,sin\frac{\beta z}{2})}{\square}\right\}\right]$$

or $\varrho_1 = \varrho_{10} + \varrho_{11} + \cdots$

$$= \left(\gamma f_{15} - \frac{f_1}{2}\,cos\,\frac{\beta z}{2} - \frac{\beta}{2} sin\,\frac{\beta z}{2}\right) + 1(\gamma f_{16} - \frac{\beta}{2}\,cos\,\frac{\beta z}{2} + \frac{f_1}{2}\,sin\,\frac{\beta z}{2}) \quad \text{.......... (1.33)}$$

$$\phi_{10} = SPr\, e^{-S\,\mathrm{Pr}\, z} \quad \text{.......... (1.34)}$$

$$\phi_{11} = C_{\phi 11}e^{-S\,\mathrm{Pr}\,z} + 1\,SPr\,(x^2+y^2)^{-1/2}\left[zS\,\mathrm{Pr}\,e^{-S\,Prz} + S^2Pr^2e^{-SPr\,z} - x\,\mathrm{Pr}\,e^{-h_1 z}\,\frac{\left\{\frac{\beta}{2}(S\,\mathrm{Pr}\,2h_1)\sin\frac{\beta z}{2} + k_2\cos\frac{\beta z}{2}\right\}}{\frac{\beta^2}{4}(S-2k_1)^2 + L_1^2}\right] \quad \text{.......... (1.35)}$$

$$\phi_{12} = C_{\phi 12}e^{-S\,\mathrm{Pr}\,z} + \left\{S\,Pr + S^3Pr^3(x^2+y^2)^{-1} + C_{111}(x^2+y^2)^{-1/2}\right\}$$
$$\{-zs\,\mathrm{Pr}\,e^{-S\,\mathrm{Pr}\,z} - S^2Pr^2e^{-S\,\mathrm{Pr}\,z}\}$$
$$+\left\{x\,Pr(x^2+y^2)^{-1/2}C_{111} + x\,S^3Pr^4(x^2y^2)^{-1} - \frac{S\,Pr^2L_4}{L_5}(x^2+y^2)^{-1}\right\}L_{11}$$
$$-\left\{y\,Pr(x^2+y^2)^{-1/2}C_{111} + y\,S^3Pr^4(x^2y^2)^{-1} - \frac{S\,Pr^2L_3}{L_5}(x^2+y^2)^{-1}\right\}L_{12}$$
$$+(x^2+y^2)^{-1}\left(z - \frac{3}{2\,SPr}\right)\frac{e^{-SPr\,z}}{2} + x\,S^2Pr^3(x^2+y^2)^{-1}L_9 - y\,S^2Pr^3(x^2+y^2)^{-1}L_{10}$$
$$+\frac{S\,Pr^3(x^2+y^2)^{-1}}{2L_5}(-xL_3+yL_4)L_{13} + \frac{SPr^3(x^2+y^2)^{-1}(-xL_4+yL_3)e^{-h_2 z}}{2L_5h_2(a+s)} - \frac{S\,Pr^3(x^2+y^2)^{-1}(xL_4+yL_3)}{2L_5}L_{14} \quad \text{......... (1.36)}$$

Where some constants and expressions f_1 , $C_{\phi 11}$, $C_{\phi 12}$, L_1 are given in appendix.

The solutions for second order term is solved by the same method are :

$$\varrho_{20} = -\frac{m_0^2}{2}e^{-m_0 z} \quad \text{........(1.37)}$$

$$\varrho_{21} = C_{22}\, e^{-(S+\alpha+1\beta)^{z/2}} + \frac{1m_0^2}{2}(x^2+y^2)^{-1/2}\left[\frac{2\,e^{-m_0 z}}{S(S+\alpha+1\beta)} + 2x\, e^{-k_1 z\left\{\frac{L_1 \sin\frac{\beta z}{2} - \frac{\beta}{2}(S-2k_1)\cos\frac{\beta z}{2}}{\frac{\beta^2}{4}(S-2k_1)^2+L_1^2}\right\}}\right]$$

$$+\frac{1m_0(x^2+y^2)^{-1/2}e^{-k_1 z}}{2\left\{\frac{\beta^2}{4}(S-2k_1)^2+L_1^2\right\}}\left[xS + x\alpha + y\beta\left\{\frac{\beta}{2}(S-2k_1)\sin\frac{\beta z}{2+L_1}\cos\frac{\beta z}{2}\right\} + (x\beta - y\,S - y\alpha)\left\{L_1\sin\frac{\beta z}{2} - \frac{\beta}{2}(S-2k_1)\cos\frac{\beta z}{2}\right\}\right] \quad \text{......... (1.38)}$$

$$\phi_{20} = \frac{s^2 Pr^2}{2} e^{-S\,\mathrm{Pr}\,z} \quad \text{....... (1.39)}$$

$$\phi_{21} = C_{\phi 21} e^{-S\,\mathrm{Pr}\,Z} - -1\,S^2Pr^2(x^2y^2)^{-1/2}\left[-2zs\,\mathrm{Pr}\,e^{-SPr\,z} - 2S^2Pr^2e^{-SPr\,z} + 2x\,\mathrm{Pr}\,e^{-h_1 z}\left\{\frac{\frac{\beta}{2}(SPr-2h_1)\sin\frac{\beta z}{2+k_2}\cos\frac{\beta z}{2}}{\frac{\beta^2}{4}(SPr-2h_1)^2+k_2^2}\right\} - 2y\,\mathrm{Pr}\,e^{-h_1 z}\left\{\frac{k_2\sin\frac{\beta z}{2} - \frac{\beta}{2}(SPr-2h_1)\cos\frac{\beta z}{2}}{\frac{\beta^2}{4}(S\,Pr-2h_1)^2+k_2^2}\right\}\right] \quad \text{............ (1.40)}$$

The constants C_{22} , $C\phi_{21}$ are obtained by same method.

RESULTS

Primary flow and temperature distribution is displayed in fig. (1-5). For the velocity component U_0 , we observe that the velocity increases as we increase as we increase suction parameter S or Rotational parameter R_{OT} or both. But the pattern for V_0 is different. It increases with the increase of suction parameter S. This is in conformity with the physical situation that the rotational parameter increases the velocity V_0. The pattern is reversed when we decrease the parameter S to a negative number see fig. 4. As for ϕ_0 is concerned, it decrease appreciably with the increase of suction parameter S and Prandt1 number Pr. The thermal layer thickness also decreases with increase of S or Pr.

The secondary flow component U_1 is exhibited graphically in Fig. 6. The velocity decreases appreciably in the narrow region near the disc with the increase of suction and rotational parameter. The magnitude of velocity decreases with z but its pattern is periodic in nature as it is evident from Fig. 7. Though the contribution of U_1 to the total velocity component U is small but it remains present for large values of z. The velocity component V_1 has also the same nature, it is periodic in character and its magnitude increases with the increase of suction parameter S or rotational parameter R_{OT} fig. 8-9. The variables ϕ_1 and ϕ_2 the first order and second order perturbations in temperature are shown in fig. 10-11. Their effects lie in a narrow region and this region further contracts as we increase the suction parameter S or Prandt1 number Pr.

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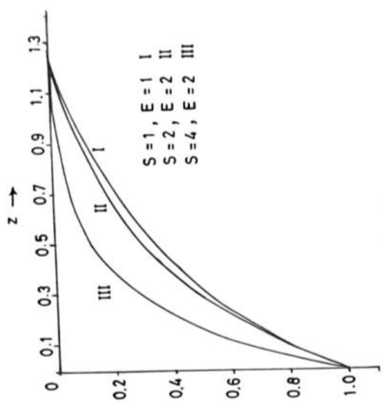


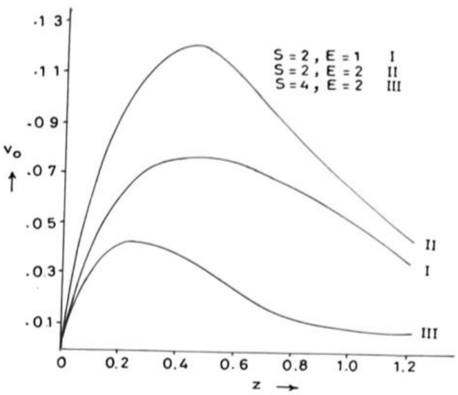
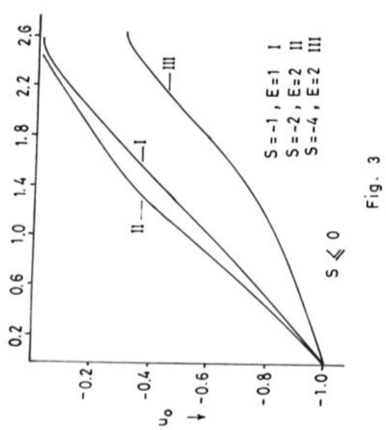
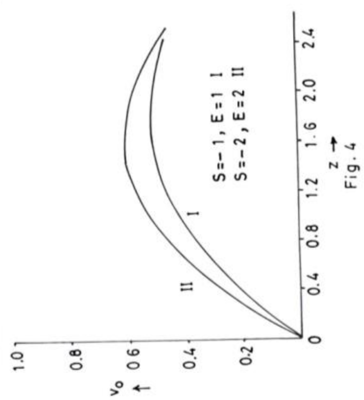


Fig. 2





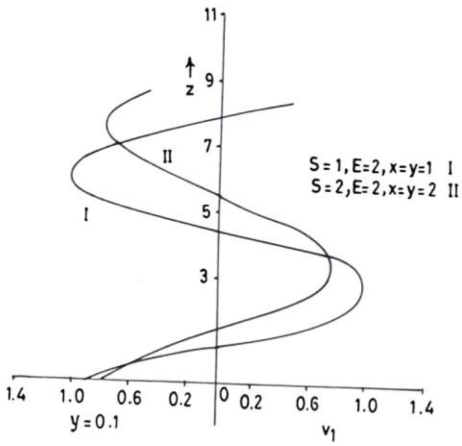


Fig. 9

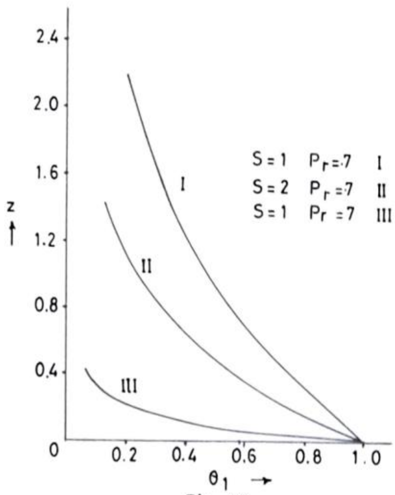


Fig. 10

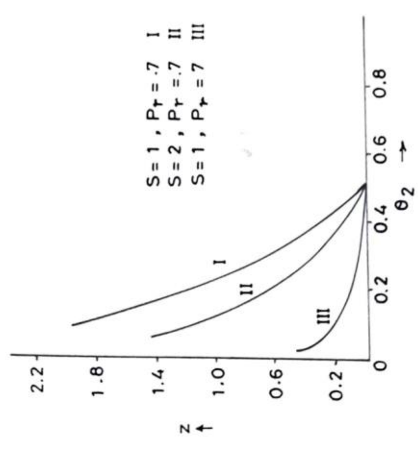


Fig. 11