SKIN FRICTION COEFFICIENT AND NUSSELT NUMBER IN FLUID FLOW AND HEAT TRANSFER BETWEEN TWO VERTICAL PLATES-II

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INTRODUCTION

I studied the problem of three-dimensional free convective heat transfer between two long vertical plates, moving in opposite directions with equal velocities. The cosinusoidal suction velocity as a function of spanwise component z is applied normal to the plate and temperature of the downward moving plate is taken spanwise consinusoidal. The governing equations are solved using regular perturbation technique. The obtained solutions are functions of dimensionless parameters Grashof number, Prandtl number and cross-flow Reynolds number. The expressions for skin friction coefficient and the rate of heat transfer at both the plates are also obtained and numerical values are presented in the form of tables for various combination of these parameters.

PROBLEM FORMULATION

The three-dimensional flow of a viscous incompressible fluid between two long vertical plates at a distance 'd' apart. The plates are moving in opposite directions with equal velocity U_0^*. Let $x^* - axis$ be along the plate in vertical upward direction and $y^* - axis$ normal to the plate. The x^* *and* z^* axis are in the plane of the plate at $y^* = 0$.

Consider the suction velocity at $y^* = 0$ as

$$v^* = -V_0\left(1+\epsilon\ cos\frac{\pi\, z^*}{l}\right) \text{ .... (1)}$$

and the temperature of the downward moving plate as

$$T^* = T_d + (T_s - T_d)\left(1+\epsilon\ cos\frac{\pi\, z^*}{l}\right) \text{ .... (2)}$$

where 'l' is wavelength, T_d the constant temperature of the plate at $y^* = d$ *and* T_s is some constant temperature. The plates are large compared to distance between them. All physical variables are assumed to be independent of x^*. If (u^*, v^*, w^*) be the velocity components in x^*, y^*, z^* direction respectively, then Navier-Stokes equations and energy equation are

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \text{ ....(3)}$$

$$v^*\ \frac{\partial u^*}{\partial y^*} + w^*\ \frac{\partial u^*}{\partial z^*} = \upsilon\ \left[\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right] + g\ \beta\ (T^* - T_d) \text{ ....(4)}$$

$$v^*\ \frac{\partial v^*}{\partial y^*} + w^*\ \frac{\partial v^*}{\partial z^*} = \upsilon\ \left[\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right] \text{ ....(5)}$$

$$v^*\ \frac{\partial w^*}{\partial y^*} + w^*\ \frac{\partial w^*}{\partial z^*} = \upsilon\ \left[\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right] \text{ ....(6)}$$

$$v^*\ \frac{\partial T^*}{\partial y^*} + w^*\ \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p}\ \left[\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right] \text{ ....(7)}$$

where k is thermal conductivity and β is volumetric coefficient of thermal expansion of the fluid.

The boundary conditions are:

$$y^* = 0, u^* = -U_0^*, v^* = -V_0\ \left(1+\epsilon\ cos\ \frac{\pi z^*}{1}\right),\quad w^* = 0,$$

$$T^* = T_d + (T_s - T_d)\ \left(1+\epsilon\ cos\frac{\pi z^*}{1}\right)$$

$$y^* = d, u^* = U_0^*, v^* = -V_0, w^* = 0, T^* = T_d \text{ .....(8)}$$

Now let us introduce following dimensionless variables

$$x = \frac{x^*}{d}, y = \frac{y^*}{d}, z = \frac{z^*}{d}, u = \frac{u^* d}{\upsilon}, v = \frac{v^* d}{\upsilon}, w = \frac{w^* d}{\upsilon},$$

$U_0 = \frac{U_0^* d}{v}, \theta = \left(\frac{T^* - T_d}{T_s - T_d}\right)$ ………….(9)

The corresponding boundary conditions are :

$$y = 0, \left| \begin{array}{l} u = -U_0 \; v = -R(1+\in \cos \pi \lambda z) \\ w = 0, \;\; \theta = 1+\in \cos \pi\lambda z \end{array} \right.$$

$y = 1, u = U_0, v = -R, w = 0, \theta = 0$ ……(10)

And the involved dimensionless parameters are :

$R = \frac{V_0 d}{v}$ (cross flow Reynold number)

$Pr = \frac{\mu C_p}{k}$ (Prandtl number) $\lambda = {}^{d}/_{l}$

$G = g \frac{\beta d^3 (T_s - T_d)}{v^2}$ (Grashof number) ……..(11)

SKIN-FRICTION COEFFICIENT AND NUSSELT NUMBER

The skin-friction coefficient at both the plates are given by

$C_{fx} = \frac{\tau_x}{\frac{\mu v}{l^2}} = real \left(\frac{\partial u}{\partial y}\right)_{y=0, y=1}$ ………….. (12)

$C_{fx}\big|_{y=0,y=1} = \left[-C_7 R e^{-Ry} - \frac{C_3 G e^{-R\,\mathrm{Pr}\,y}}{R(1-\mathrm{Pr})}\right] + \in \cos \pi\lambda z$

$[m_1 C_{13}e^{m_1 y} + m_2\, C_{14}e^{m_2 y} - G\{D_5\, m_3 e^{m_3 y} + D_6\, m_4 e^{m_4 y}$

$+D_7\, R\, \mathrm{Pr}\, e^{-RPry} + D_8(m_1 - R\,\mathrm{Pr})\, e^{(m_1 - R\,\mathrm{Pr})\,y}$

$+ D_9\, (m_2 - R\,\mathrm{Pr})\, e^{(m_2 - R\,Pr)y}\} - \pi\lambda\, \{-RD_{10}\, e^{-Ry}$

$- D_{11}(m_1 - R)\;\; e^{(m_1 - R)y} - D_{12}(m_2 - R)\, e^{(m_2 - R)y}$

$-D_{13}\, RPr\, e^{-R\,Pr} - D_{14}\, e^{(m_1 - R\,Pr)y} \times (m_1 - R\,Pr)$

$-D_{15}(m_2 - R\,Pr)\, e^{(m_2 - R\,Pr)y}]$ ……..(13)

The dimensionless coefficient of heat transfer i.e. Nusselt number are given by

$Nu = -\frac{q}{\rho\, V_0\, C_p\, (T_s - T_d)} = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0, y=1}$ ……(14)

$Nu\big|_{y=0,y=1} = -[-R\,\mathrm{Pr}\, C_3 e^{-RPry} + \in \cos \pi\lambda z$

$\{C_{11}\, m_3 e^{m_3 y} + m_4\, C_{12}\, e^{m_4 y} - C_3 R\, \pi\, \lambda\, Pr^2\, (-D_1\, R\, Pr\, e^{-RPry}$

$+D_2(m_1 - R\,Pr)e^{(m_1 - R\,Pr)y} + D_3\,(m_2 - R\,Pr)e^{(m_2 - R\,Pr)y})\}]$ ….(15)

The skin-friction coefficient and Nusselt number at both the plates $y = 0\; and\; y = 1$ for various combination of parameters (Table-I) are given in Table II, III, IV, V.

TABLE -I: Various Combinations Of Parameters

Case	R	Pr	G	λ	∈
I	-2	0.7	5	0.1	0.1
II	2	0.7	5	0.1	0.1
III	20	0.7	5	0.1	0.1
IV	40	0.7	5	0.1	0.1
V	2	7	5	0.1	0.1
VI	2	7	5	0.3	0.1
VII	2	0.7	10	0.1	0.1
VIII	2	0.7	15	0.1	0.1
IX	2	0.7	5	0.1	0.3

TABLE-II:Skin friction coefficient C_{fx} vs $\pi\lambda z$ Case at the plate y = 0

Case/ $\pi\lambda z$	I	II	III	IV	V	VI	VII	VIII	IX
$\pi\lambda z = 0$	-68.30085	28.1908	29.57813	33.5	0.52856	0.7406	55.90772	83.6233	78.23267
$\pi\lambda z = \frac{\pi}{4}$	-48.05112	20.68433	22	25.625	0.6131249	0.7615	40.89807	61.11109	55.71388
$\pi\lambda z = \frac{\pi}{2}$	.-8016782	3.169748	4.357143	8.17852	0.8104	0.8104	5.87689	8.584038	3.169748
$\pi\lambda z = \frac{3\pi}{4}$	46.44776	-14.34485	-13.28125	-9.25	1.0077	0.85929	-29.14429	-43.943	49.37378
$\pi\lambda z = \pi$	66.69748	-21.8512	-20.85938	-17	1.0923	0.8802318	-44.15381	-66.45533	-71.89307
$\pi\lambda z = \frac{5\pi}{4}$	46.44776	-14.34485	-13.28125	-9.25	1.0077	0.85929	-29.14429	-43.943	49.37378

Table – III: Skin friction coefficient C_{fx} vs $\pi\lambda z$ Case at the plate y = 1

Case/ $\pi\lambda z$	I	II	III	IV	V	VI	VII	VIII	IX
$\pi\lambda z = 0$	216.9141	-8.81339	-5.2$\times 10^{-5}$	3.097548$\times 10^{-5}$	6.086136$\times 10^{-2}$	5.897142 $\times 10^{-2}$	-17.68726	-26.56104	-24.8367
$\pi\lambda z = \frac{\pi}{4}$	152.7906	-6.409867	-3.6$\times 10^{-5}$	2.168284$\times 10^{-5}$	.0585892	5.72672$\times 10^{-2}$	-12.88088	-19.35181	-17.62622
$\pi\lambda z = \frac{\pi}{2}$	3.169749	-.8016785	-6.8224$\times 10^{-5}$	-2.8880633$\times 10^{-13}$	5.32908$\times 10^{-2}$	5.329088 $\times 10^{-2}$	-1.665964	-2.5302249	-.8016785
$\pi\lambda z = \frac{3\pi}{4}$	-146.451	4.8065	3.54341$\times 10^{-5}$	-2.168284$\times 10^{-5}$	4.799199$\times 10^{-2}$	0.0493145	9.548935	14.29132	16.02289
$\pi\lambda z = \pi$	-210.5745	7.209976	5.091256$\times 10^{-5}$	3.097548$\times 10^{-5}$	4.572058$\times 10^{-2}$	4.761033 $\times 10^{-2}$	14.35532	21.50055	23.23337
$\pi\lambda z = \frac{5\pi}{4}$	-146.451	4.806504	3.543415$\times 10^{-5}$	-2.168284×10^{-5}	4.799199$\times 10^{-2}$	.0493145	9.548935	14.29132	16.02289

Table – IV; Nusselt number vs $\pi\,\lambda\,z$ Case at the plate y = 0

Case/ $\pi\,\lambda\,z$	I	II	III	IV	V	VI	VII	VIII	IX
$\pi\,\lambda\,z = 0$	.4736943	2.130306	16.72501	33.6	16.775	16.7796	2.130306	2.130306	2.674446
$\pi\,\lambda\,z = \frac{\pi}{4}$	.4690566	2.048684	15.90751	31.92	15.94251	15.94573	2.048684	2.048684	2.429583
$\pi\,\lambda\,z = \frac{\pi}{2}$	.4582352	1.858235	14.00001	28.	14.00001	14.00001	1.858235	1.858235	1.85235
$\pi\,\lambda\,z = \frac{3\pi}{4}$	.4474139	1.667786	12.09251	24.08	12.05751	12.0543	1.6677864	1.6677864	1.286888
$\pi\,\lambda\,z = \pi$	.4427762	1.586165	11.27501	22.4	11.22501	11.22042	1.586165	1.586165	1.042024
$\pi\,\lambda\,z = \frac{5\pi}{4}$	.4474139	1.667786	12.09251	24.08	12.05751	12.0543	1.6677864	1.667786	1.286888

Table – V: Nusselt number vs $\pi\,\lambda\,z$ / Case at the plate y = 1

Case/ $\pi\,\lambda\,z$	I	II	III	IV	V	VI	VII	VIII	IX
$\pi\,\lambda\,z = 0$	2.093573	.4843903	5.919363×10^{-6}	1.89801×10^{-12}	6.712105×10^{-6}	4.00475×10^{-6}	.4843903	.4843903	0.5367005
$\pi\,\lambda\,z = \frac{\pi}{4}$	2.022972	.4765438	7.635976×10^{-6}	7.13699×10^{-12}	8.190896×10^{-6}	0.295748×10^{-6}	.4765438	.4765438	0.5131609
$\pi\,\lambda\,z = \frac{\pi}{2}$	1.858235	.4582352	1.164141×10^{-5}	1.936031×10^{-11}	1.64141×10^{-5}	1.164141×10^{-5}	.4582352	.4582352	0.4582352
$\pi\,\lambda\,z = \frac{3\pi}{4}$	1.693409	.4399266	1.564684×10^{-5}	3.158392×10^{-11}	1.509192×10^{-5}	1.698707×10^{-5}	.4399266	.4399266	0.4033095
$\pi\,\lambda\,z = \pi$	1.622897	.4320801	1.736346×10^{-5}	3.682261×10^{-11}	1.657071×10^{-5}	1.927807×10^{-5}	.4320801	0.4320801	0.3797699
$\pi\,\lambda\,z = \frac{5\pi}{4}$	1.693409	.4399266	1.564684×10^{-5}	3.158392×10^{-11}	1.509192×10^{-5}	1.698707×10^{-5}	.4399266	.4399266	0.4033095

RESULTS

The skin-friction coefficient and rate of heat transfer i.e. Nusselt number at both the plates y = 0 and y = 1 for various combination of parameters Table I and given in Table II, III, IV and V.

The coefficient of skin friction C_{fx} in the x-direction at y = 0 increases with the increase of Reynolds number R, decrease of Prandtl number Pr and increase of λ. It also increases with the increase of Grashof number G and parameter $\in$, Table II. The coefficient of skin friction at y = 1 decreases with the increase of Reynold number R, decrease of Prandtl number Pr and increase of λ. C_{fx} also decrease with the increase of Grashof number G and parameter ∈ , Table III.

The rate of heat transfer i.e. Nusselt number Nu from the plate at y = 0 increases with the increase of Reynold number R, an increase of Prandtl number and parameter ∈ see Table IV. The Nusselt number Nu from the plate at y = 1 decreases with the increase of R, Pr, ∈ , see Table V. There is no effect of Grashof number G on rate of heat transfer at both the plates situated at y = 0 and y = 1.

REFERENCES

Bansal 2013: - Fluid flow and heat transfer between two vertical plates moving in opposite direction international research journal of science engineering and technology, RJSET vol 3 issue 1. And references therein.